Indian Statistical Institute, Bangalore

B. Math. Second Year

Second Semester - Computer Science II

Back Paper Exam | Duration: 3 hours | Date : May 25-29, 2015

Answer all the questions. Max Marks: 50

1. In Octave/Matlab what do **eps, realmin** and **realmax** represent. What is *overflow* and *underflow* ? [5]

2. Let $\boldsymbol{A}$ be an $n \times n$ with $det\boldsymbol{A} \neq 0$. State and prove conditions under which $\boldsymbol{A}$ can be decomposed uniquely as
$$\boldsymbol{A} = \boldsymbol{LU}$$
where $\boldsymbol{L}, \boldsymbol{U}$ are lower and upper triangular matrices. [10]

3. Define a matrix norm and explain how it differs from a vector norm . Show that the vector norm
$$\|x\|_\infty = \max_i |x_i|$$
for $x \in \mathbb{R}^n$ induces the matrix norm [10]
$$\|\boldsymbol{A}\|_\infty = \max_i \sum_j |a_{ij}|.$$

4. Let $\boldsymbol{A}$ be the matrix
$$\begin{bmatrix} \epsilon & 0 & 0 & 1 \\ 0 & \epsilon & 0 & 0 \\ 0 & 0 & \epsilon & 0 \\ 1 & 0 & 0 & \epsilon \end{bmatrix}$$
where $\epsilon \neq 0$. Write down the $\boldsymbol{LU}$ decomposition of this matrix. [15]

5. We wish to solve the equation
$$\sin(x) = x^2$$
One solution is at $x = 0$, but we are interested in finding the other solution $x^\star \neq 0$ Write down an iteration formula for Newton's method for solving this problem, i.e an expression for x_k in terms of x_{k-1}. [10]